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# The Design, Construction, and Testing of an Exhaust-gas Thermoelectric Generator

Vernon D. Pepper

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**THE DESIGN, CONSTRUCTION, AND TESTING  
OF AN EXHAUST-GAS THERMOELECTRIC  
GENERATOR**

**BY**

**VERNON D. PEPPER**

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**A thesis submitted  
in partial fulfillment of the requirements for the  
degree Master of Science, Department of  
Agricultural Engineering, South Dakota  
State College of Agriculture  
and Mechanic Arts**

**March, 1961**



26670

**THE DESIGN, CONSTRUCTION, AND TESTING  
OF AN EXHAUST-GAS THERMOELECTRIC  
GENERATOR**

This thesis is approved as a creditable, independent investigation by a candidate for the degree, Master of Science, and acceptable as meeting the thesis requirements for this degree; but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

**Thesis Advisor**

**Head of the Major Department**

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V.D.P.

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## INTRODUCTION

Thermoelectricity is the direct conversion of heat energy to electrical energy, or vice versa. Conventional electrical generating systems convert heat energy by one or more intermediate processes to mechanical energy, which in turn is used to drive a generator. Each intermediate process has an associated energy loss or inefficiency which accumulates until the over-all conversion efficiency may range from less than 1 per cent to about 40 per cent depending on the system used and the amount of energy produced (9)\*. It is reasonable to assume that elimination of the intermediate processes by use of thermoelectricity could reduce the heat energy required per unit of electrical energy developed. Until recently, however, the efficiency of conversion by thermoelectric means using metals was so low (less than one per cent) that little serious thought was given to such uses. As a result, the primary use of thermoelectricity has been for temperature measurement and control with thermocouples, where the low efficiency was of little importance.

In recent years the development of improved semiconductor materials has led to renewed interest in the possibility of thermoelectric power generation competing economically with mechanical systems of power generation. These new materials have already improved the conversion efficiency tenfold (2) and promise much more in the future. Since the efficiency of a thermoelectric generator is

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\*Numbers in parenthesis refer to appended references.



essentially unaffected by its size or power output, the first take-over by thermoelectric generators is expected to be in the low power-producing region where conventional mechanical systems are least efficient. Figure 1 graphically represents the present and future expected areas of competition.

One area in which thermoelectricity could first become economically competitive is where otherwise waste heat could be utilized to produce electrical energy, and first cost rather than efficiency could be the controlling factor. There are many power generating systems in use today which emit considerable amounts of heat as waste. One of these is the internal combustion engine, which expels from the exhaust system about one-third of its total heat generated (1). If this heat could be utilized by a thermoelectric generator to recharge the engine's battery or as an auxillary power source for portable electrically operated equipment, the output of that engine could be increased at very little or no additional operating cost.

It is acknowledged that the thermoelectric materials for such a generator are not economically competitive at the present time. Based upon future expectations of highly efficient and low-cost materials, however, a preliminary investigation of requirements and limitations of such a system appeared justified.

The purpose of this project was to (a) make a preliminary investigation of some of the design principles involved in developing an exhaust-gas thermoelectric generator, (b) construct a low-power generator utilizing the design principles established, and (c) test the

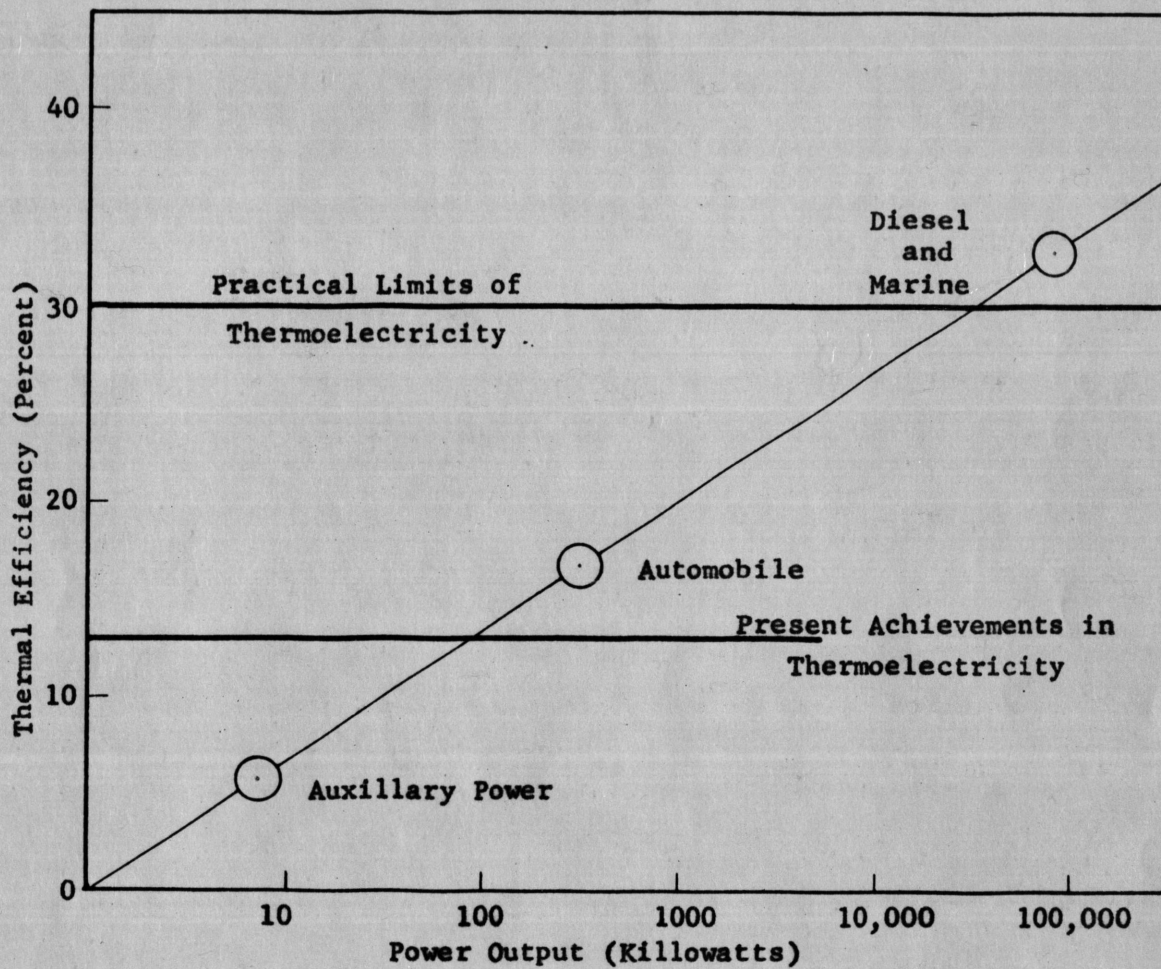


Figure 1. Graphical Comparison of Thermal Efficiency and Power Output Between Thermoelectric Devices and Conventional Power-Producing Systems

(from Sommers and Kelly-11)

generator under actual operating conditions to verify or disprove the design and the expected performance. It is also highly desirable that the generator be adaptable to new thermoelectric materials as they are developed. In that way a compatible generator design could be available for the new materials when they become available for use in commercial thermoelectric devices.



## THE DISCOVERY AND DEVELOPMENT OF THERMOELECTRICITY

Thermoelectricity was first discovered by Thomas Seebeck in 1822. He showed that an electric current flowed in a closed circuit of two dissimilar metals when the junctions of the two metals were maintained at different temperatures. The open-circuit voltage producing this flow of current was called the Seebeck emf (electromotive force). It was later shown that the Seebeck emf was a combination of several electromotive forces.

Jean Peltier in 1834 showed that a current flowing through the same type of circuit as that used by Seebeck would cool one junction and heat the other, and that the process was reversible. He then related the Peltier emf at each junction to the amount of energy released or absorbed as heat. This energy was found to be proportional to the first power of the current, whereas Joule heating, which is irreversible and always evolving heat, is proportional to the square of the current.

Another factor about the Seebeck emf which was noted by William Thompson in 1854 was that the Seebeck emf was not proportional to the first power of the temperature difference. Actually, the Seebeck emf increases to a maximum, drops off, and even becomes negative with increasing temperature difference of the junctions in an approximate parabolic arc. Thompson explained this fact by stating that another emf must be present in the circuit due to the temperature gradient in each individual metal. It can be shown that the temperature gradient along a metal rod is changed when a current passes through the rod, and

that this change in gradient is reversible. The energy associated with this phenomenon is called the Thompson emf, which is distributed along the rod according to the temperature gradient.

The Seebeck emf, then, is a combination of two Peltier emf's and two Thompson emf's, all of which are dependent not only on the absolute temperature but also on the temperature gradient (7).

As has been previously stated, the efficiency of conversion of heat to electricity by the Seebeck effect is less than one per cent for metals. For this reason very little importance was placed on such a means of power production until recent years when improved semiconductor materials were developed with efficiencies of conversion of 10 per cent and more, due mainly to the much larger Seebeck emf which occurs in such materials. These materials are primarily 2 and 3-element compounds, doped to obtain the desired characteristics.

Doping is a process by which the number of free electrons in the molecular structure is controlled by varying the amount of the basic materials and impurities in the compound. Doping also provides reversed polarity in certain materials. This is brought about by a flow of positively charged "holes" vacated by electrons, thereby permitting pairs of elements to be arranged electrically in series but thermally in parallel. The materials with excess electrons are called N-type materials, and those with holes are called P-type materials (5).

An oversimplification of electron theory is necessary to grasp the complexity of the interrelationships between the Seebeck emf, the electrical resistivity, and the thermal conductivity. These three

factors, along with the ability of the material to withstand high temperatures, control the effectiveness of any material for use in thermoelectric power generation.

Consider a simple thermocouple made up of an N-type and a P-type thermoelement arranged as shown in Figure 2. The following discussion will deal directly with the N-type thermoelement, but will apply equally as well to the P-type, remembering that the current flow and the emf are reversed from that in the N-type material.

Within a thermoelectric material, the Seebeck emf arises from a flow of electrons toward the cold end and away from the hot end when a temperature gradient exists. As the electrons accumulate, the cold end becomes negatively charged and begins to repel the approaching negative charges until a balance exists between the electrons arriving and those being repelled, with the cold end remaining negatively charged. The fewer the electrons available for the return flow, the higher the negative charge which can exist and the higher the Seebeck emf (5). However, the fewer the number of free electrons the higher the electrical resistivity within the material. Also, the thermal conductivity is reduced by fewer free electrons, but to a lesser extent since heat is conducted by the lattice structure of the material as well.

It can readily be seen there is an optimum number of free electrons above which the Seebeck emf is insufficient and below which the electrical resistivity is too great for any appreciable power output. This optimum occurs at approximately  $10^{19}$  electrons per cubic centimeter, which is in the range of semiconductor materials (6).



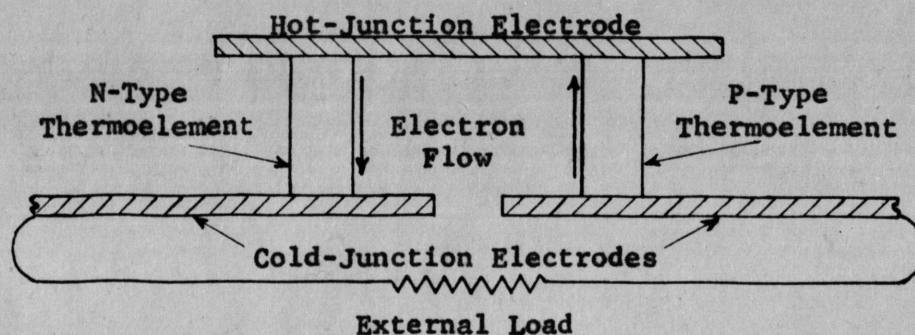
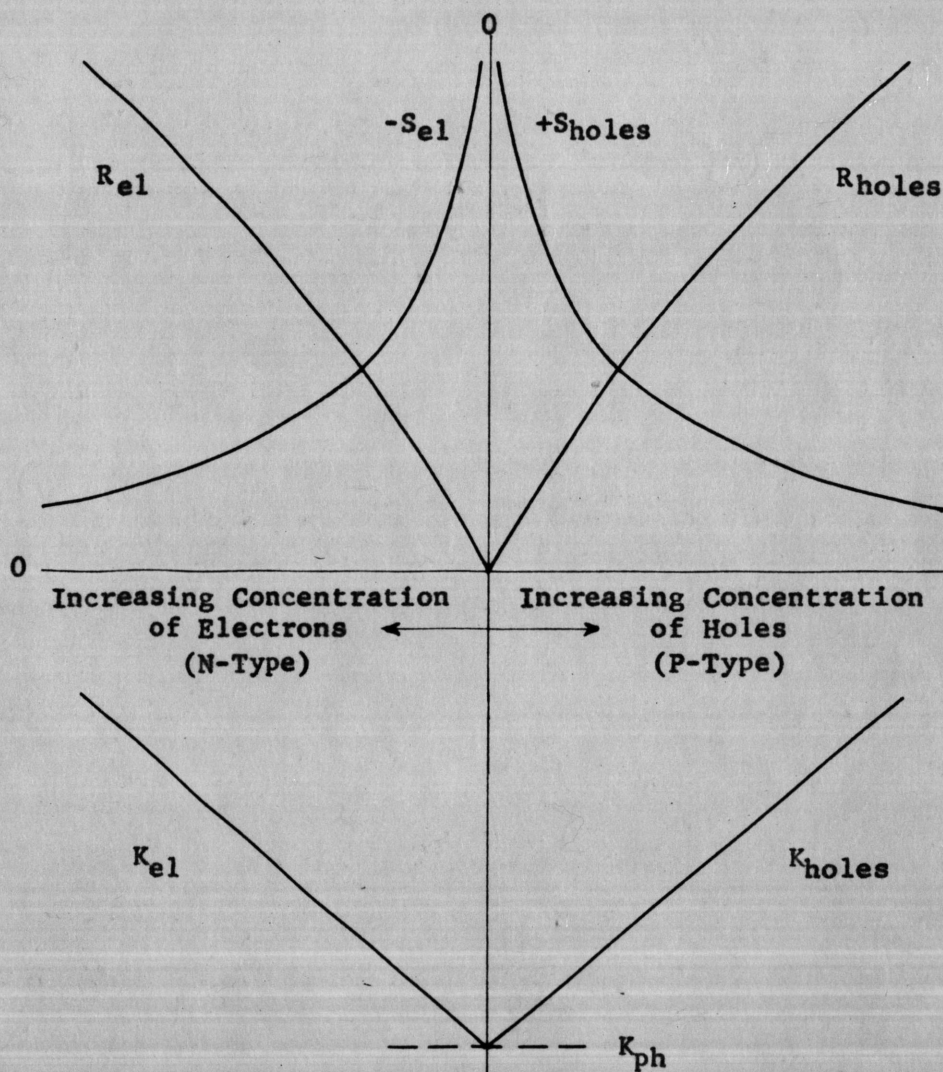


Figure 2. A Simple Thermocouple



(from Jaumot-6)

Figure 3. Curves Showing the Approximate Dependence of the Seebeck Emf, the Electrical Resistivity, and the Thermal Conductivity on the Free Electron Concentration

Figure 3 shows the effect of the number of electrons or holes on the Seebeck emf (S), the electrical resistivity (R), and the thermal conductivity (K) where K is  $K_{el}$  (due to electron concentration) plus  $K_{ph}$  (due to lattice structure).

The commonly called "figure of merit" which is used by many investigators to evaluate thermoelectric materials arrives from the dependence of S, R, and K on the electron concentration. The figure of merit is defined as

$$Z = \frac{S^2}{RK}$$

where

Z = figure of merit (per degree)

S = Seebeck emf (volts per degree)

R = Electrical resistivity (ohm-inches)

K = thermal conductivity (watts per degree per inch).

A value for Z of about  $0.6 \times 10^{-3}/^{\circ}\text{F}$  is generally considered minimum for practical thermoelectric generator materials.

Thermoelectric generators have been constructed on an experimental and evaluation basis by a number of organizations and manufacturers. Celent (4) has a resume' of companies working on thermoelectric materials and devices, a table showing the materials developed, and a table showing generators which have been constructed.

Of the generators constructed to date, the heat source ranges from combustion of liquid or gaseous fuels, to solar heat, to nuclear heat. The cooling method ranges from liquid cooling, to radiation, to natural or forced air convection. The power output varies from less



than 1 watt to 5000 watts. The oldest known generator was put in operation in 1955; therefore, no long term information is available on expected life of a thermoelectric generator. Semiconductors are inherently stable materials, however, and the generator constructed in 1955 is still performing satisfactorily.

There is no evidence of anyone having constructed or even having planned a thermoelectric generator to use on the exhaust system of an internal combustion engine, or more specifically, agricultural tractors. Celent (4) has a very elementary sketch of a thermoelectric generator using automobile or aircraft exhaust heat and liquid cooling which he suggests as a possible application of thermoelectricity, but apparently nothing was done to develop the idea.

The only known commercially available high temperature (above 600°F) thermoelectric elements are lead telluride (PbTe) produced by Minnesota Mining and Manufacturing Company. Because of the limited variety of materials and shapes of the elements, there is no need to present the many and complex formulas and equations involved in design and analysis of the various materials with thermoelectric properties. If a further background of information is desired by the reader there are many references listed in the bibliography which will furnish additional material on composition, affects of impurities, and other governing data concerning production of thermoelectric materials. The matter of importance in this study was to use the materials available and to apply the information given by the manufacturer concerning their physical characteristics to obtain a satisfactory generator design.

## DESIGN CONSIDERATIONS

The design of an exhaust-gas thermoelectric generator involved the following distinct but interrelated considerations: thermoelectric material selection, element orientation and generator shape, material selection for the generator body, types of hot and cold junctions, material selection for the hot- and cold-junction electrodes, heat absorption at the hot junctions, and heat dissipation at the cold junctions. All of these separate considerations were interrelated in that the failure of any one portion or section would mean failure of the entire generator, and any limitations imposed by one material or operating condition would affect the requirements and operating conditions of other materials. For example, the cooling media had to carry off whatever heat leaked through the generator body as well as what passed through the elements, and the hot-junction temperature dictated the element and the generator-body material. An over-all picture had to be maintained even when considering only one small portion of the entire generator design problem.

### Thermoelectric Element Selection

Selecting the proper thermoelectric materials for an exhaust-gas generator involved three main aspects: determination of the temperature range involved, finding materials which would withstand the temperature, and evaluation of the available materials in terms of shape, number of elements, and thermoelectric power to determine the expected power output.

### Temperature Range Involved

The maximum temperature which the thermoelements would have to withstand was determined by taking exhaust-gas temperature readings of a diesel tractor and a gasoline tractor by use of thermocouples. Figure 4 shows the exhaust-gas temperatures of each tractor plotted against the per cent of maximum horsepower at rated pto (power-take-off) speed. These curves can be considered typical for each type of engine, with other engines varying as much as  $\pm 10$  per cent. For example, the maximum exhaust temperature of the gasoline engine could be changed as much as 250°F by adjusting the fuel-air mixture and the ignition timing. Since the engines tested were not operating at maximum economy or maximum power throughout the range of adjustments, some engines could conceivably be operating at exhaust temperatures of up to 1400°F. All temperatures were measured in the exhaust pipe approximately two feet beyond the muffler. Additional temperature measurements at other than rated pto speed are included in Appendix A.

It is clearly shown in Figure 4 that a thermoelectric generator output would not be uniform since the exhaust-gas temperature varies with the load on the engine. This is also true of a conventional engine-driven tractor generator, but with the engine speed controlling the variation. In both cases a voltage regulator could compensate for the nonuniformity.

The exhaust-gas temperature varies over a wider range in the diesel engine and has less potential for producing thermoelectric power than in the gasoline engine, especially under part-load conditions. If



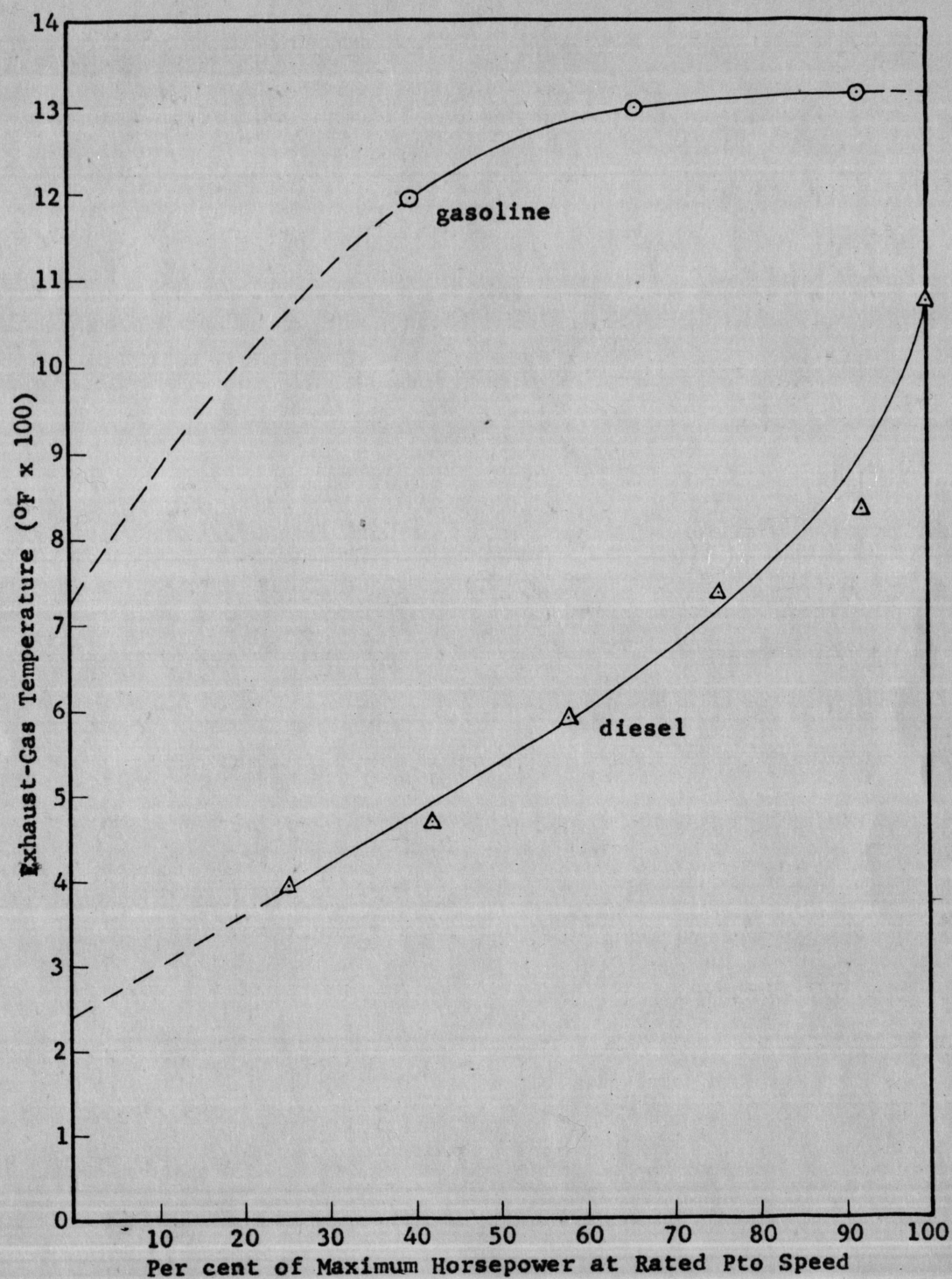


Figure 4. Graph of Exhaust-Gas Temperature Versus Per Cent of Maximum Horsepower at Rated Pto Speed for Gasoline and Diesel Engines

the present trend toward greater use of diesel power for agricultural purposes continues, the potential value of thermoelectric exhaust-gas generators may decrease in this field.

### Thermoelectric Material Selection

An operating temperature of 1100°F to 1200°F placed a severe restriction on the number of available thermoelectric materials which could be used. Minnesota Mining and Manufacturing Company produces P- and N-type lead telluride thermoelectric elements which will withstand temperatures up to 1100°F. These elements are sold for experimental and prototype evaluation only. Six sizes of elements are available as shown in Table I. Table II gives the general physical characteristics as determined by the manufacturer. The manufacturer also supplies curves of resistivity and Seebeck emf versus temperature as shown in Figures 5 and 6. These characteristics were sufficient to calculate the expected power output, maximum current, current at maximum power, and thermal efficiency.

### Evaluation of Thermoelement Performance

Using the data from Table I and assuming the maximum recommended hot-junction temperature of 1100°F and a cold-junction temperature of 100°F, the figure of merit for a thermoelectric couple would be

$$Z = \frac{S^2}{RK}$$

$$Z = \left( \frac{0.240V}{1000^\circ F} \right)^2 \left( \frac{1}{(0.00272 \text{ ohm-in}) (0.0282 \text{ watts/in } ^\circ F)} \right)$$

$$Z = 0.75 \times 10^{-3} \text{ } ^\circ F^{-1}$$

TABLE I. ELECTRICAL PROPERTIES OF P-TYPE AND N-TYPE THERMOELECTRIC ELEMENTS  
PRODUCED BY MINNESOTA MINING AND MANUFACTURING COMPANY

Element Dimensions Diameter x Length (in.)	L/A Per Element (in <sup>-1</sup> )	Power to Matched Load Per Couple (watts)	Watts/lb Per Couple (watts)	Cost/Element	
				N-Type (\$)	P-Type (\$)
1/2 x 1	5.09	1.04	8.95	11.45	12.27
1/2 x 1/2	2.54	2.09	36.0	6.80	7.32
1/4 x 1	20.4	.260	8.95	5.47	5.77
1/4 x 1/2	10.2	.520	36.0	3.88	4.10
1/4 x 1/4	5.09	1.04	144.0	3.15	3.27
3/16 x 1	36.2	.146	8.95	4.91	5.17

Hot-junction temperature = 1100°F

Cold-junction temperature = 100°F

Average resistance = (0.00272) L/A per couple

Seebeck emf = 0.240 volts per couple

(from 3M Co.-8)

TABLE II. PHYSICAL PROPERTIES OF P-TYPE AND N-TYPE THERMOELECTRIC ELEMENTS  
PRODUCED BY MINNESOTA MINING AND MANUFACTURING COMPANY

Physical Property	P-Type	N-Type
Color Dot on Element	Red	Blue
Machineability	Not Advisable	Not Advisable
Operating Temperature	1100°F	1100°F
Resistivity ( $\pm 10\%$ ) at 75°F	150 Micro-Ohm in.	200 Micro-Ohm in.
Standard Dimensional Tolerance	$\pm 0.005$ in.	$\pm 0.005$ in.
Tensile Strength	>1000 psi	>1000 psi
Compression Strength	>10,000 psi	>10,000 psi
Young's Modulus	$2 \times 10^6$ psi	$2 \times 10^6$ psi
Density	8.15 g/cc	8.15 g/cc
Thermal Expansion Coefficient	$18 \times 10^{-6}/^\circ\text{C.}$	$18 \times 10^{-6}/^\circ\text{C.}$
Thermal Conductivity Approximate Value 100-1100°F	.02 watts/cm/°C.	.02 watts/cm/°C.

(from 3M Co.-8)



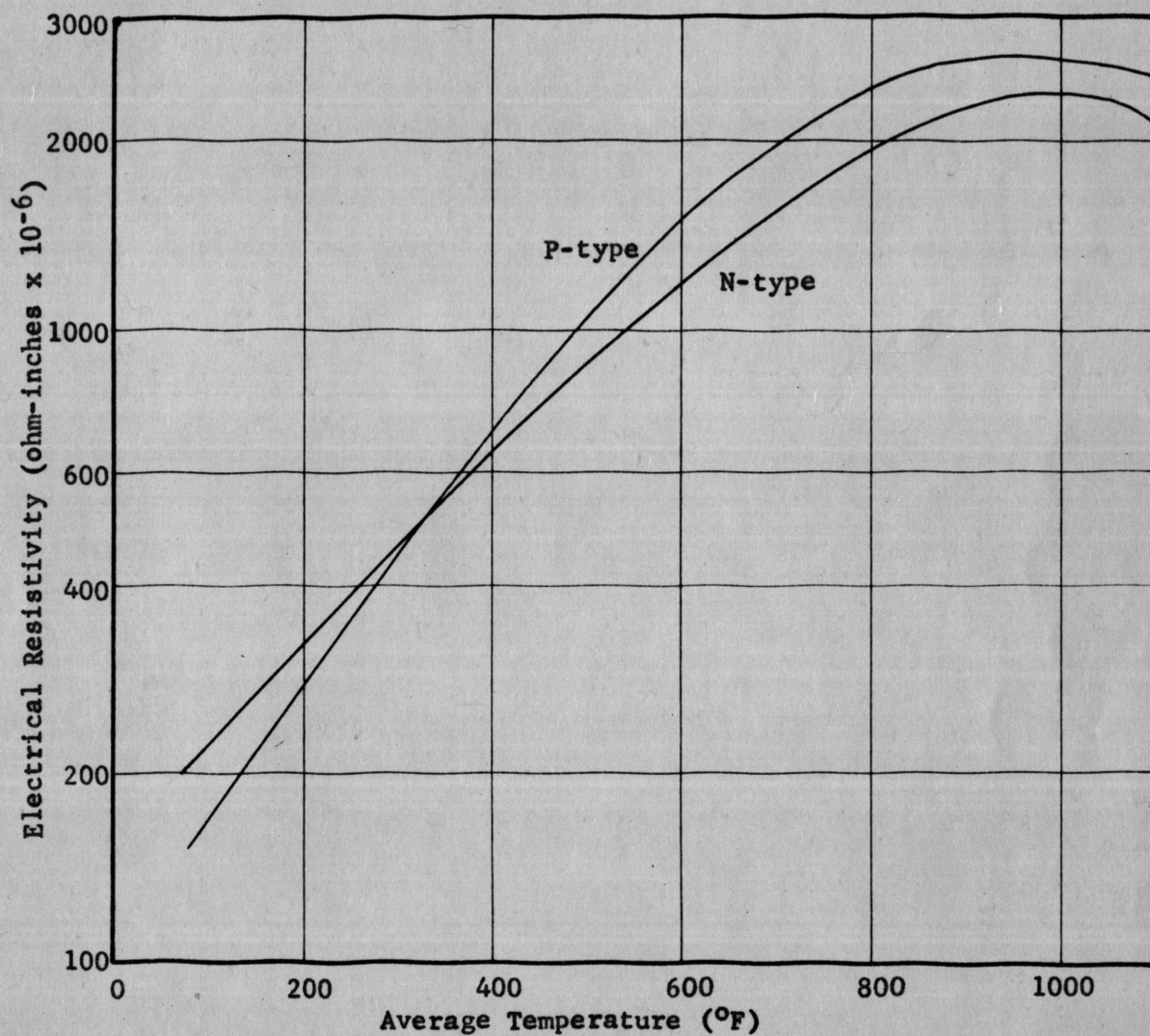


Figure 5. Graph of Electrical Resistivity Versus Average Temperature for P and N-type Lead Telluride Thermoelements

(from 3M Co.-8)



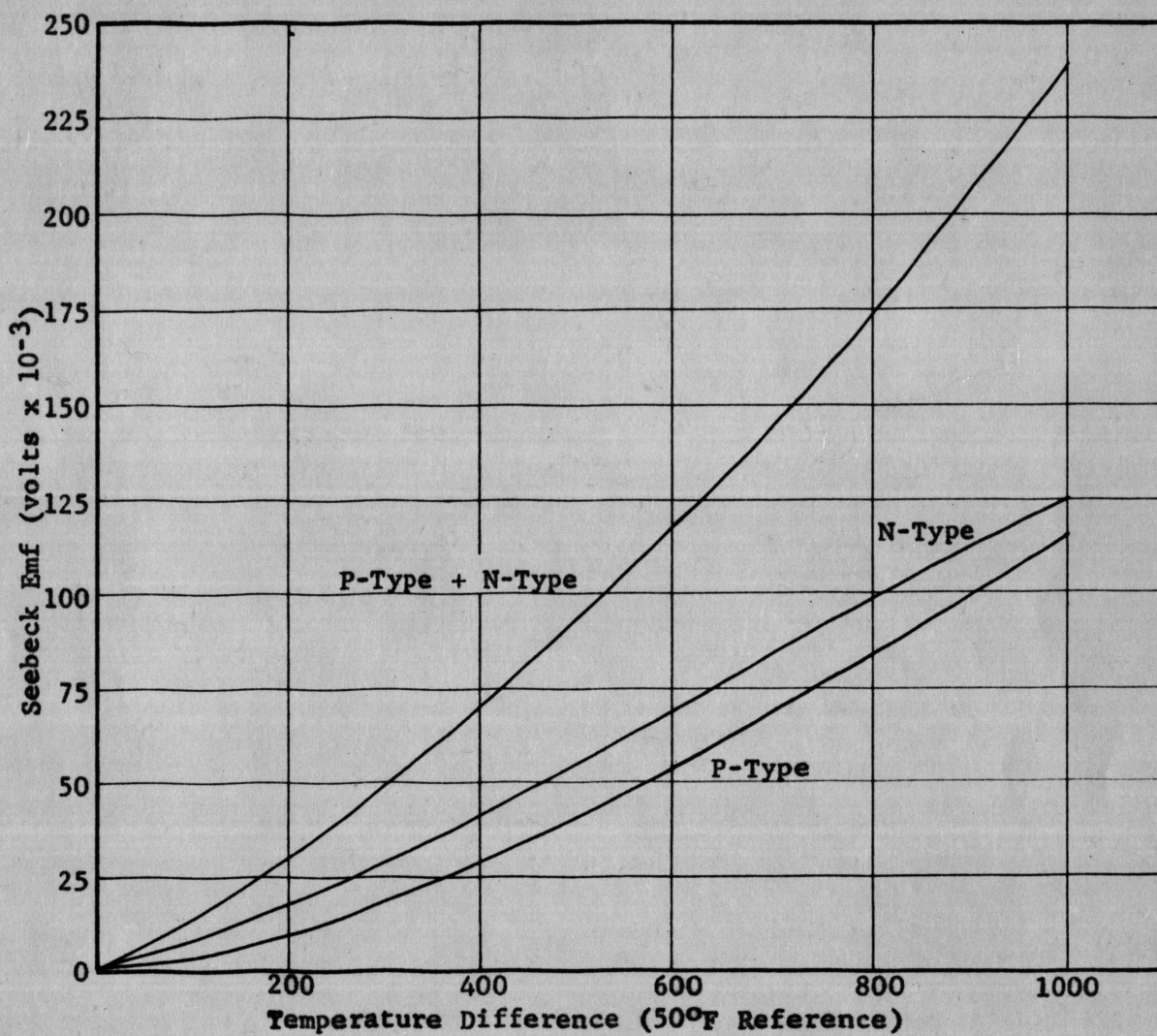


Figure 6. Graph of Seebeck Emf Versus Temperature Difference for P and N-type Lead Telluride Thermoelements

(from 3M Co.-8)

For a preliminary investigation of this sort, the desired current, voltage, and power were arbitrary. The procedure involved in determining these factors in an actual design problem, however, could follow the same outline as used here.

Assuming a maximum desirable short-circuit current ( $I$ ) of 10 amps, the internal resistance ( $R$ ) would be

$$R = \frac{V}{I}$$

where  $V$  is the total Seebeck voltage over the temperature range involved and  $R$  is the internal resistance of the thermoelements plus the contact resistances plus the connecting electrode resistances. If the only significant internal resistance was that of the thermoelements (which should occur in a good generator design), the internal resistance would be 0.00272 ohm-inches  $\times$   $L/A$  per inch (from Table I), and the desired  $L/A$  would be

$$0.00272 \text{ ohm-in } (L/A \text{ in}^{-1}) = \frac{0.240 \text{ volts}}{10 \text{ amps}}$$

$$L/A = 8.83 \text{ in}^{-1}.$$

Also from Table I, the 1/4-inch-diameter by 1/2-inch-long elements had the nearest available  $L/A$  of 10.2 per inch. The maximum current from these elements was

$$I = \frac{0.240 \text{ volts}}{0.00272 \text{ ohm-in } (10.2 \text{ in}^{-1})} = 8.64 \text{ amps.}$$

The required number of couples was arrived at by dividing the desired voltage by the Seebeck emf per couple. Ten couples producing a combined Seebeck emf of 2.40 volts was decided upon as sufficient for an initial design analysis.

The current for 10 couples remained the same, and the total element resistance was

$$R = 0.00272 \frac{\text{ohm-in}}{\text{couple}} \times 10 \text{ couples} \times 10.2 \text{ in/in}^2$$

$$R = 0.277 \text{ ohm.}$$

Table III was constructed by use of the previous calculations to determine the current and optimum external resistance to produce the maximum external power. The maximum power of 5.16 watts occurred at a current of 4.3 amps and a voltage of 1.20 volts. Note that the maximum power was developed at one-half the short-circuit current and one-half the open-circuit voltage. This fact can be used to determine the necessary number of couples to produce any specified useful power.

The thermal efficiency of the elements was determined by the ratio of the electrical energy produced to the total heat energy supplied to the elements. The heat (Q) transferred through the elements to maintain the temperature difference was

$$Q = KA \frac{dt}{dl} = K \left( \frac{t_h - t_c}{L/A} \right)$$

$$Q = 0.0282 \frac{\text{watts}}{\text{in } ^\circ\text{F}} \left( \frac{1000^\circ\text{F}}{10.2/\text{in}} \right) 20 \text{ elements}$$

$$Q = 55.3 \text{ watts.}$$

An additional 5.16 watts had to be supplied to replace that converted to electrical energy, and another 5.16 watts had to be supplied to replace the heat absorbed at the hot-junctions and liberated at the cold-junctions by the Peltier effect of the current flowing in the circuit (Peltier emf = IV). Thus the thermal efficiency ( $E_t$ ) was

$$E_t = \frac{5.16 \text{ watts (100)}}{(55.3 + 5.16 + 5.16) \text{ watts}} = \frac{5.16}{65.6} (100) = 7.85\%.$$

TABLE III. CALCULATIONS OF EXPECTED POWER OUTPUT FROM THE EXPERIMENTAL  
THERMOELECTRIC GENERATOR AT VARIOUS EXTERNAL LOAD SETTINGS

Current I (amp)	Internal Resistance R (ohm)	Seebeck Emf S (volts)	Internal Voltage Drop IR (volts)	External Voltage Drop S - IR (volts)	External Power I (S-IR) (watts)
1	0.28	2.40	0.28	2.12	2.12
2	0.28	2.40	0.56	1.84	3.68
3	0.28	2.40	0.84	1.56	4.68
4	0.28	2.40	1.12	1.28	5.12
4.3	0.28	2.40	1.20	1.20	5.16
5	0.28	2.40	1.40	1.00	5.00
6	0.28	2.40	1.68	0.72	4.32
7	0.28	2.40	1.96	0.44	3.08
8	0.28	2.40	2.24	0.16	1.28
8.6	0.28	2.40	2.40	-----	-----



To estimate the total amount of heat which was available in the exhaust stream, a 40-horsepower engine burning gasoline at the rate of 0.1 gal/bhp-hr with a heat value of 125,000 BTU/gal was assumed (1).

$$\text{Total heat} = 125,000 \text{ BTU/gal} \times 40 \text{ bhp} \times \frac{0.1 \text{ gal}}{\text{bhp-hr}}$$

$$\text{Total heat} = 500,000 \text{ BTU/hr.}$$

If one-third of the total heat were expelled from the exhaust pipe as previously stated (discounting muffler and manifold heat losses), the heat in the exhaust would be 167,000 BTU/hr. The heat used by a 5.16-watt generator was 65.6 watts or 225 BTU/hr. Even a 500-watt generator would use only about 13 per cent of the total available heat.

#### Element Orientation and Generator Shape

The shape of the generator body was determined after considering several factors. First, it was anticipated that maintaining the desired temperatures at the cold junctions would be more difficult than at the hot junctions since there was such a large excess of heat available. A cylindrical arrangement with the hot junctions on the inside surface compensated for this by providing the greatest ratio of cooling area to heating area of the practical shapes considered. Second, a cylindrical shape for the hot surface required no transition from the present cylindrical shape of the exhaust pipe. Finally, the heat density over a cylindrical surface was expected to be more uniform than with any other practical shape. For these reasons a cylindrical generator shape was chosen, although it probably presented more

problems in construction than a rectangular shape, especially when considering thermal stress.

There were 20 elements, 10 N-type and 10 P-type, 1/4-inch in diameter and 1/2-inch long, to be mounted in pairs such that they were thermally in parallel and electrically in series. This was accomplished by mounting them perpendicular to the exhaust pipe so they extended radially outward with the hot junctions on the inside and the cold junctions on the outside.

#### Material Selection For The Generator Body

The material used in the generator had to meet several stringent requirements. Due to the brittle nature of the thermoelements, the generator body had to provide the mechanical support for the elements and the connecting electrodes. It had to hold the components in position even though there was considerable vibration on the exhaust pipe. The material for the generator body had to be machinable to cut it to the desired shape, to inset the thermoelements, and to attach the junction electrodes and cooling apparatus. The material had to be electrically insulating to prevent charge leakage between the elements and thermally insulating to maintain the hot-junction temperature and to reduce the amount of heat which had to be dissipated by the cooling system. Finally, the material for the generator body had to be stable at high temperature, not only to maintain its mechanical strength, but also to prevent any chemical reaction from occurring between it and the thermoelements which might impair the performance of the elements.

Of the materials considered, the one which best suited the many requirements was asbestos cement board. It proved to be adequate in strength although being somewhat brittle, excellent in machining qualities, highly adequate as an electrical insulator, stable at 1200°F, and sufficiently inert chemically to reduce the possibility of contamination of the thermoelements. The thermal conductivity is 0.43 BTU/hr ft °F, which places it in the class of thermal insulators.

#### Types of Hot and Cold Junctions

Perhaps the most critical part of the entire generator design was to obtain good thermal and electrical contact between the elements and their connecting electrodes. The junctions had to offer a negligible electrical resistance as compared to the element resistance and could not retard the flow of heat into and out of the thermoelements.

As recommended by the manufacturer, a pressure contact was used for the hot junctions. The manufacturer also listed several requirements as prerequisites to use of pressure contacts for hot junctions. They were: approximately 100 psi compressive loading on the thermoelements, a junction operating temperature above 700°F to destroy the oxide film, and a non-oxidizing atmosphere to prevent additional oxide formation in the junctions.

A soldered cold junction was used, again as recommended by the manufacturer of the elements. To aid in accomplishing this, the thermoelements were pre-coated on the cold junction with a tin-bismuth solder by the manufacturer before shipment.

### Material Selection for the Hot- and Cold-Junction Electrodes

The material for the hot- and cold-junction electrodes had to (a) provide thermal conductivity into and out of the thermoelements, (b) provide electrical conductivity from one element to its companion element to form each couple, and (c) be chemically compatible with the thermoelements so as not to contaminate the thermoelements and reduce their effectiveness.

Both the hot- and the cold-junction electrodes were cut in such a fashion as to cover as much area as possible to aid conduction of heat into and away from the thermoelements while still providing separation between adjoining couples.

The hot-junction electrodes had to be electrically insulated from the exhaust pipe to prevent short circuiting of the electrodes through the pipe. This was accomplished by spreading a thin coating of Allen AL-Pl cement on the exhaust pipe in the area of contact between the electrodes and the pipe. This cement provided the electrical insulation necessary without materially reducing the heat transfer rate, and the cement is stable to at least 1400°F.

The material for the hot-junction electrodes was the most critical from the contamination standpoint, and the manufacturer of the elements recommended low alloy steels as the most compatible material for use at high temperatures. Low alloy steels were also satisfactory for heat and electrical conductivity provided they had a sufficiently large cross-sectional area to give no appreciable resistance to current flow.



Copper or brass was selected as possible material for the cold-junction electrodes because of its high thermal and electrical conductivity. Spring brass could also be used to provide the compressive loading required of the hot junctions, thereby serving two functions. For this reason spring brass was used for the cold-junction electrodes.

#### Heat Absorption at the Hot Junctions

Since the exhaust-gas temperature could be as high as 1400°F and the maximum recommended temperature for the thermoelements was 1100°F, it was anticipated that it would not be difficult to transfer the required amount of heat through the exhaust-pipe wall and the hot-junction electrodes. To attempt to predict the temperature drop which might occur required several assumptions which limited the accuracy of the calculations.

The calculations involved in determining the exhaust velocity, the film coefficient, and the hot-junction area required are shown in Appendix B. A 1.875-inch inside diameter exhaust-pipe section required 2.25 inches of length (0.667 square inches per element) for sufficient area to transfer the required 225 BTU/hr with a 300°F temperature drop between the exhaust gas and the hot-junction electrodes. The calculated exhaust velocity was 19,150 fpm.

A convenient arrangement of ten thermoelements (five couples) each in two rows, with the rows spaced one inch apart was used, even though it resulted in 10 per cent less (0.59 square inches per element) than the calculated required area.

### Heat Dissipation at the Cold Junctions

The two logical means of dissipating the heat from the cold end of the thermoelements were with circulating water and with natural or forced air convection. The circulating water method appeared to present more problems in fabrication and could result in a heavier and more cumbersome generator. With natural air convection the cooling fin area required was expected to result in an excessively large generator. Preliminary considerations of forced air cooling suggested a light, small, and simple generator. This is due mainly to two desirable characteristics of the particular application of thermoelectric generators to agricultural tractor exhaust systems.

One characteristic in favor of air cooling in general was that the tractor would be operated primarily out-of-doors. In the winter months the cold out-door air would provide the most efficient cooling, which corresponds to the time that the load on the electrical system is greatest. Secondly, forced air convection in particular was readily provided by use of the high exhaust velocity and the jet-pump principle to draw cooling air over the fins. For these reasons forced air cooling promised to provide the most satisfactory method of dissipating the heat from the cold end of the thermoelements.

The calculations involved in determining the heat load on the cooling fins, the cooling air velocity available, and the size and number of cooling fins required are shown in Appendix C. The heat to be dissipated by the cooling fins was 653 BTU/hr. The hood and venturi arrangement as shown in Figure 7 provided a cooling air flow of a maxi-

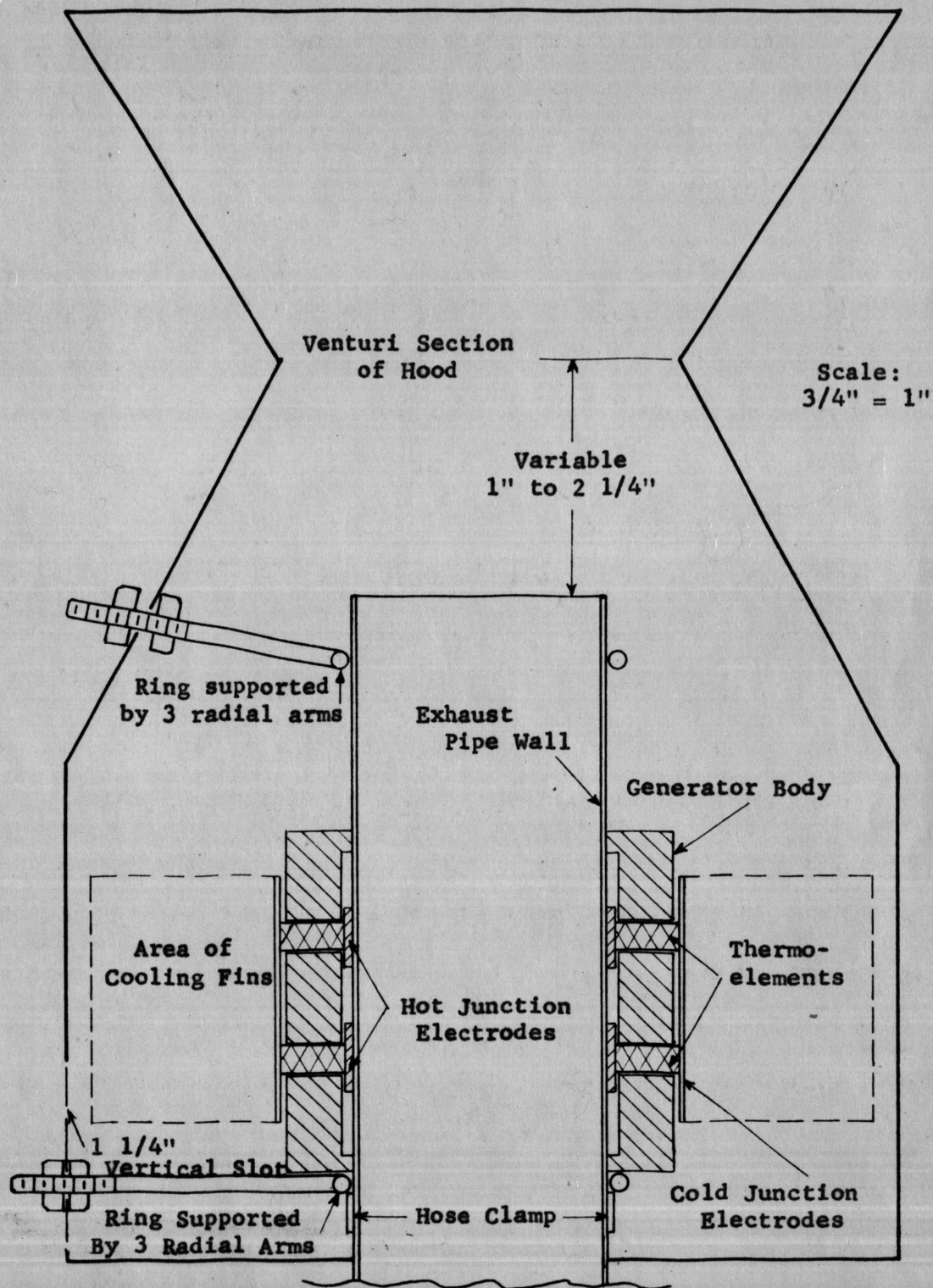


Figure 7. A Sectioned Sketch of the Exhaust-Gas Generator and the Cooling Hood Used for Forced Air Cooling

num of 2000 fpm. The number of 1 1/2-inch-long by 1-inch-wide fins required under these circumstances was determined to be 43 fins, allowing a drop of 50°F between the average fin temperature and the cooling air throughout the temperature range involved.



## PROCEDURE

The procedure used to determine the merit of the design considerations was to construct a generator using the design principles established and to test the generator under actual operating conditions. During the tests data were collected on heat transfer into and out of the generator, temperatures at the thermoelement junctions, and current and voltage output of the generator.

### Generator Assembly

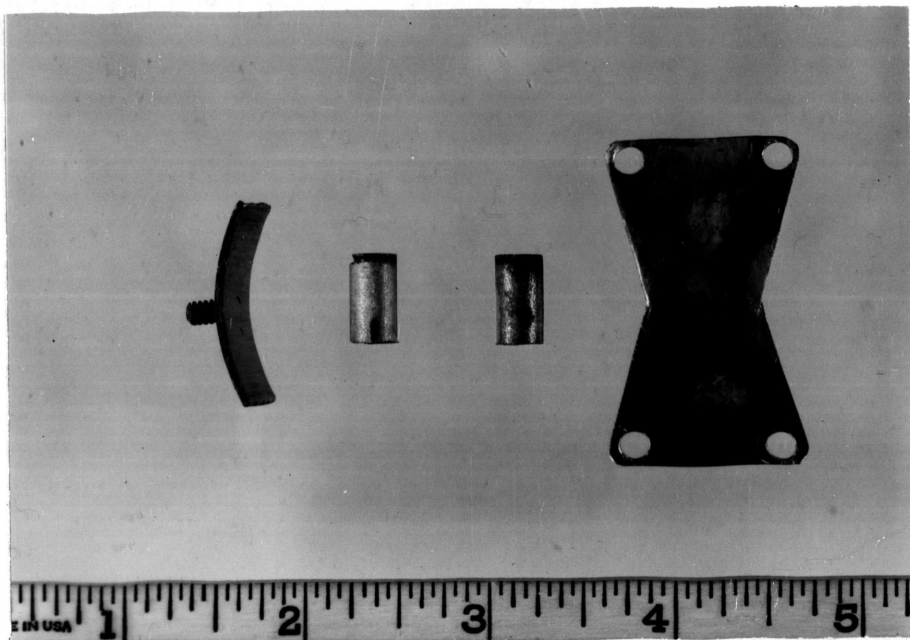
The thermoelectric generator was assembled using the design principles established.

A 6-inch length of 2-inch exhaust pipe was obtained. The center three inches of its outside surface was prepared for the application of AL-P1 cement by polishing the surface to bare metal and roughening with a coarse file. The ceramic cement was brushed on to a thickness of about 0.003 to 0.005 inches. The pipe was then placed in an oven to dry the recommended one hour at 200°F and one hour at 600°F with a maximum temperature increase of 200°F per hour between the two drying temperatures.

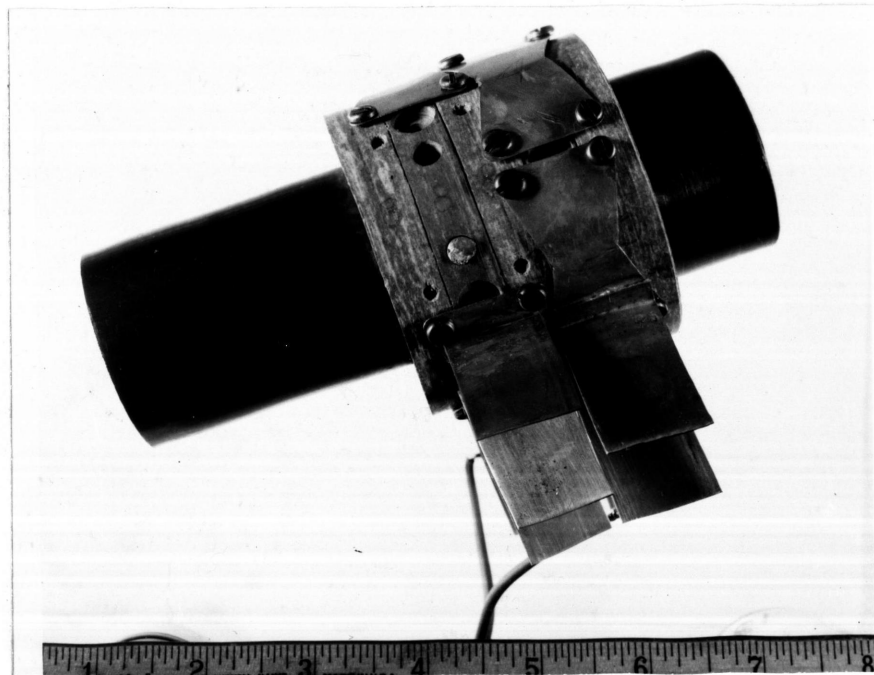
Five doughnut-shaped rings of 2 1/4 inches inside diameter and 3 inches outside diameter were cut from 1/2-inch-thick asbestos cement board. Two of these rings were each drilled with ten 1/4-inch-diameter holes extending radially through the rings to receive the thermoelements. The two drilled rings were located alternately between the other three rings which served as spacers and to hold the cold-junction electrodes.

The hot-junction electrodes were made of iron straps  $1\frac{1}{4}$ -inch long by  $\frac{1}{2}$ -inch wide by  $\frac{1}{16}$ -inch thick and were cold formed to fit the pipe contour. Guide pins were made by placing size 6-32 machine screws through holes drilled in the center of the straps. The screw heads were welded to the strap, and the weld was ground flush. Figure 8 shows a finished hot-junction electrode. The guide pins served to hold the electrodes in place until the thermoelements were in place and to allow the electrodes to move in and out as the thermal stress changed. Mating holes of  $\frac{1}{8}$ -inch diameter were drilled through the asbestos cement board to receive the guide pins. The face of the electrodes upon which the thermoelements rested was made flat and perpendicular to the thermoelement by turning a  $\frac{1}{4}$ -inch spot facer with the pilot removed through the thermoelement hole and against the electrode. The spot facer marked the area and plane of contact, which was then filed smooth to enlarge the area and to remove the hump left in the center by the absence of the pilot.

The cold-junction electrodes were constructed of 0.018-inch-thick spring brass. The shape of the electrodes as shown in Figure 8 was such that they provided as much area as possible upon which to mount fins, but prevented contact between adjacent electrodes. Figure 9 shows the method of alternating the electrodes. This shape of electrode was also chosen to allow fastening of the electrodes to the spacer rings rather than the already weakened thermoelement rings. The electrodes were fastened to the generator body with  $\frac{1}{4}$ -inch-long size 6 tapping screws. The holes in the electrodes were drilled  $\frac{1}{16}$ -inch



**Figure 8. From Left to Right, A Hot-Junction Electrode, a P and an N-type Element, and a Cold-Junction Electrode**



**Figure 9. Partially Assembled Thermoelectric Generator**

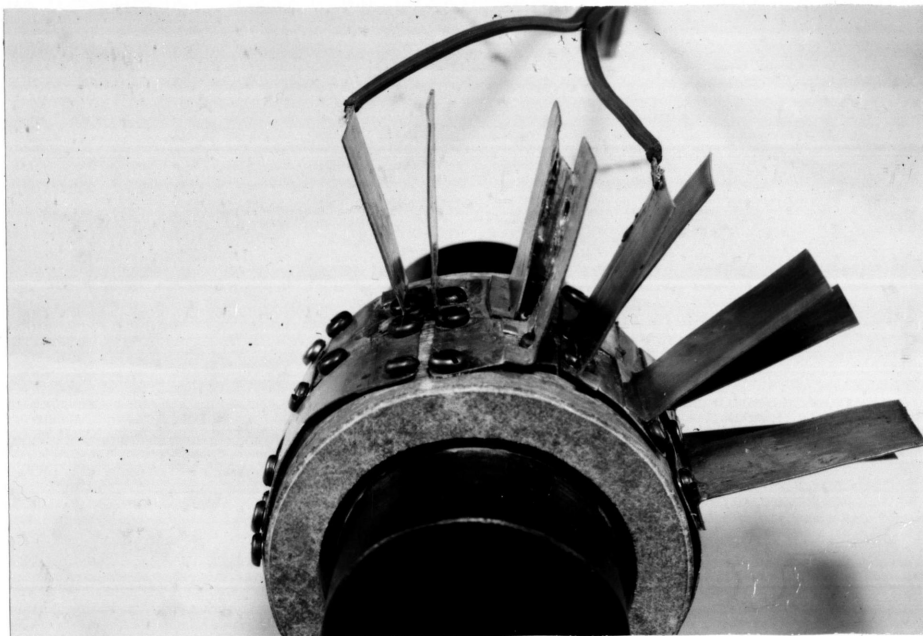
over-size to allow for thermal expansion and bending of the spring brass. The electrodes were tinned on the under-side with a tin-bismuth solder in preparation for joining to the thermoelements.

To assemble the generator, the hot-junction electrodes were placed in position in the rings and the rings were slipped over the pipe section in proper order. The elements were dropped into their receiving holes with the tinned end exposed. The N-type and P-type elements were alternated around each ring and also across the rings. The series electrical connection was completed by placing the cold-junction electrodes over pairs of elements which alternated with the pairs connected on the hot-junction side.

The electron flow around the circuit was through an N-type element from hot to cold, across the cold-junction electrode, through the adjacent P-type element from cold to hot, and across the hot-junction electrode to the next N-type element in the row. The two rows of elements were connected in series by cutting two of the cold-junction electrodes as shown in Figure 10 and connecting the N-type and the P-type elements of the cut electrodes together by bridging across the fins. Lead wires were soldered to the fins of the other halves of the separated electrodes as terminals for attaching an external load.

The cooling fins of 1 1/2-inch-long by 1-inch-wide by 0.022-inch-thick copper were soldered to the electrodes with a solder-flux paste while the electrodes were fastened loosely over the thermoelements. The heat applied to solder the fins also joined the thermoelements and the electrodes by melting the solder at the





**Figure 10. Method of Connecting Rows of Thermoelements to Continue the Series Arrangement of Electron Flow**



**Figure 11. Over-all View of the Thermoelectric Generator Mounted on a Tractor Exhaust Pipe**

junctions. Sixty fins were soldered to the cold-junction electrodes, even though it was determined that 43 would be sufficient. This was done to improve the efficiency of the cooling method, possibly to the point that natural air cooling would be sufficient. The electrodes were then pressed down on the thermoelements by tightening the fastening screws. This provided the compressive loading required by the hot junctions.

No method was provided to seal the hot junctions from the air for several reasons. It was desired to know if the junction resistance could be maintained at a low value if a non-oxidizing atmosphere was not provided, and to what extent the oxide film, if it did form, would affect the generator output. Secondly, observation of the operation of the junctions and the measurement of the junction temperature by thermocouples would be very difficult if the generator were to be sealed air tight.

The generator hood as shown in Figure 11 was formed from a five-quart tin cylinder which was 6 1/2 inches in diameter and 8 inches long. The venturi was formed by slitting the cylinder at 2-inch intervals one-half the length of the cylinder and overlapping the strips 1 inch, which resulted in a diameter of 3 1/4 inches at the throat. A 5 1/2-inch funnel was cut off at the point where the diameter was 3 1/8 inches, and the small diameter of the funnel was placed inside the small diameter of the cylinder so that the two edges just overlapped. The two edges were joined by crimping and soldering. The hood was held to the exhaust pipe by 2 wire rings each connected to 3 radial wire

arms of the same size which projected through the hood. The arms were threaded and double-nutted where they extended through the hood. One ring was located on the pipe above the generator and the other ring was below the generator. The upper ring was free to slide up and down on the pipe, while the lower ring was fastened between the generator and a hose clamp to hold it stationary. The arms of the lower ring passed through 1 1/4-inch slots in the hood so that the hood could be moved up and down or held in any position by the double-nuts on the lower ring. In this manner the open end of the exhaust pipe could be varied from 3/4 inch to 2 inches below the venturi throat to adjust the air flow over the cooling fins.

The generator assembly was placed on the exhaust pipe by use of a slotted sleeve and hose clamps. This method resulted in a tight fit with no observable exhaust leakage.

#### Method of Testing

The generator was tested under actual operating conditions on the exhaust pipe of two internal combustion engines. One was a 6-cylinder, 50-horsepower, gasoline stationary power unit and the other was a 4-cylinder, 38-horsepower, gasoline tractor engine. The first engine was operated indoors and was used for the first tests of the initial generator design. The second engine was operated out-of-doors and was used for tests of both the initial generator and the revised generator.

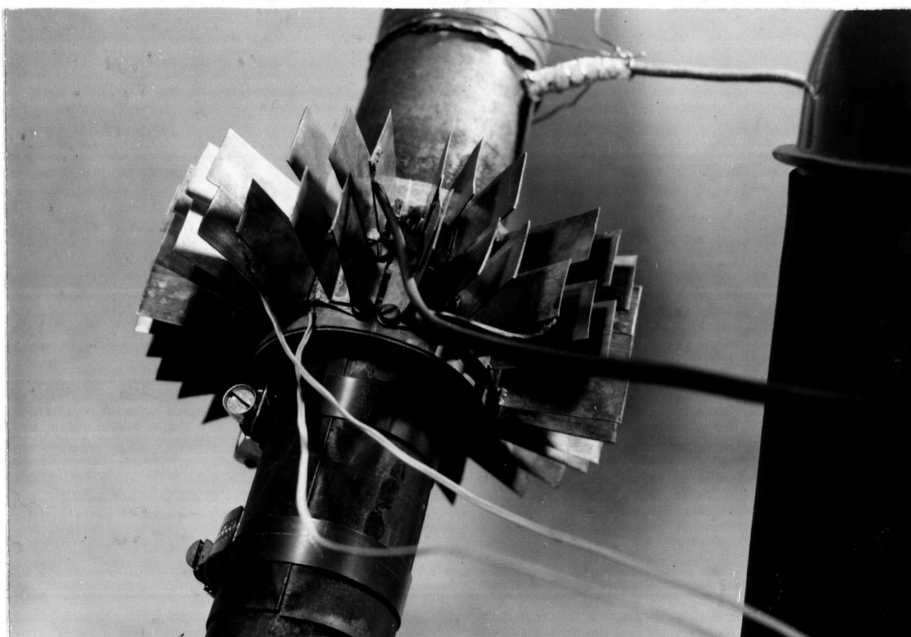
Operating temperatures were measured at various points within the generator and in the exhaust stream by thermocouples. Copper-Constantan thermocouples of 32-gage wire were soldered to a fin tip and to the underside of a cold-junction electrode to obtain the average fin temperature and the cold-junction temperature. A 32-gage wire Chromel-Alumel thermocouple welded together at the tip was placed between the hot-junction electrode and the asbestos cement board adjacent to a thermoelement to obtain the hot-junction temperature. A 22-gage wire Chromel-Alumel thermocouple welded together at the tip was used to measure the exhaust-gas temperature by placing it directly in the gas stream. Figure 12 shows the mounted generator with the thermocouples attached.

The Copper-Constantan thermocouples were connected to a single-range Leeds and Northrup potentiometer which indicated temperature directly to 350°F. A Leeds and Northrup #8662 portable precision potentiometer which reads the Seebeck voltage was used with the Chromel-Alumel thermocouples. Temperatures were obtained by reference to a conversion table and were limited only by the maximum temperature which the thermocouple material could withstand.

The air velocity obtained through the hood and across the cooling fins was measured with a hot-wire Anemotherm Air Meter anemometer.

The voltage output of the generator was measured with a resistance type volt-ohm meter. Both open-circuit voltage and voltage drops across a variable load were measured with this instrument.





**Figure 12. Close-up View of the Thermoelectric Generator With Thermocouples Attached**



**Figure 13. A Generator Test Set-up With the Instruments Involved in Collecting the Necessary Data**

A 15-amp ammeter and a 50-ohm variable resistor were used to measure the current flow under various external loads.

The engine rpm was determined with an ignition tachometer, and the torque output was measured with a wheatstone bridge instrument attached to a strain gage torque arm on a hydraulic dynamometer. This information gave the horsepower output of the engine. When the generator was mounted on a tractor exhaust in later studies, a pto dynamometer provided the load on the engine and indicated the load directly in horsepower.

Figure 13 shows a typical test set-up on the stationary power unit and the instrumentation involved in obtaining the required data.

## TEST RESULTS

Tests of the generator were conducted to determine if the following considerations were satisfactory: (a) the over-all generator design, (b) the physical strength of the generator itself, (c) the durability of the thermoelements, and (d) the electrical output of the generator. Any faults that appeared were corrected by redesigning the portion that failed and additional tests were conducted to evaluate the changes in design.

### Tests of the Initial Generator Design

The generator was first mounted on a 6-cylinder gasoline stationary power unit which developed 50 horsepower at 2000 rpm. The tests on this engine were conducted indoors and without the cooling hood to determine if natural air cooling could be sufficient with 60 fins.

The data collected from these tests were recorded in Table IV. The tests were discontinued after three trials because the tin-bismuth solder, which has a low melting point, melted at the cold junctions. These tests showed that forced convection was necessary to cool the cold junctions. Internal resistance values were not calculated for the first two trials because the amperage was too low to observe. This was expected because the hot-junction temperature was not sufficiently high to destroy the oxide film in the hot junctions. For the third trial, as shown in Table IV, the temperature was increased very rapidly and the readings were taken before the cold-junction temperature became

TABLE IV. DATA COLLECTED FROM TESTS OF THE NATURAL AIR COOLED INITIAL  
THERMOELECTRIC GENERATOR ON A STATIONARY ENGINE

Exhaust- Gas Temp. (°F)	Hot- Junction Temp. (°F)	Cold- Junction Temp. (°F)	Ambient Air Temp. (°F)	Open- Circuit Voltage V (volts)	Short- Circuit Current I (amps)	Internal Resistance $R = \frac{V}{I}$ (ohms)	Maximum		Engine Speed (rpm)	Load on Engine (hp)
							External Power $P = \frac{I}{2} \left( \frac{V}{2} \right)$ (watts)			
444	262	180	80	0.17	-----	-----	-----		800	0
642	358	250	80	0.15	-----	-----	-----		800	10
1120	872*	225*	80	1.50	3.5	0.43	1.31		1700	48.5

\*These are not stabilized temperatures



excessive. The low resistance value obtained in this test showed that the oxide film was rapidly being destroyed in the hot junctions.

The next tests of the generator were conducted out-of-doors on the exhaust pipe of a 4-cylinder, 38-horsepower, gasoline-burning, internal-combustion engine to which a pto dynamometer was attached.

Inspection of the thermoelements prior to these tests showed that several of the P-type elements were broken or at least chipped near the cold junctions. This breakage resulted in the loss of compressive loading on the hot junctions because the elements could bind in the holes and not move freely with thermal stress. A high internal resistance resulted and no appreciable current was obtained from this generator. It was decided, however, to continue to test the generator to get additional information on the heat transfer properties and the Seebeck voltage, which was not noticeably affected by the broken and chipped elements as long as a complete circuit was maintained.

The data as shown in Table V were collected under the previously described condition. The tests were conducted out-of-doors with cooler air temperatures than before and a light wind of about five mph.

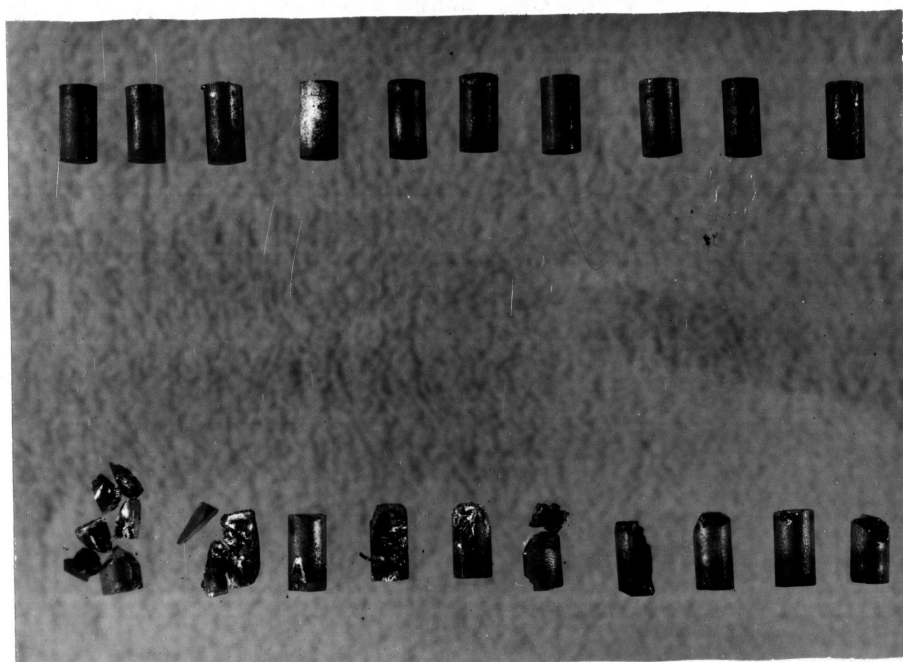
A comparison of temperatures was obtained under these conditions both with and without the cooling hood. The data showed that the hood not only decreased the cold-junction temperature, but also decreased the hot-junction temperature. This meant that the heat was being transferred through the elements and the generator body at a faster rate than it could be locally replaced. A later observation which was

TABLE V. DATA COLLECTED FROM TESTS OF THE NATURAL AND FORCED AIR COOLED  
INITIAL THERMOELECTRIC GENERATOR ON A TRACTOR ENGINE

Exhaust- Gas Temp. (°F)	Hot- Junction Temp. (°F)	Cold- Junction Temp. (°F)	Fin Tip Temp. (°F)	Cooling Air Temp. (°F)	Engine Speed (rpm)	Engine Load (hp)	Use of Hood	Velocity of Cooling Air (fpm)	Seebeck Voltage (volts)
605	334	104	84	58	1200	10	no	-----	-----
605	323	96	74	58	1200	10	yes	1000	-----
1137	899	190	120	58	1900	38	no	-----	1.6
1137	791	116	89	65	1900	38	yes	2000	1.5

made at night was that the cooling air was extracting a large amount of heat from the exposed exhaust pipe prior to reaching the generator. The muffler and about three inches of exhaust pipe above the muffler were glowing cherry red, but above that the pipe was cooler and not glowing.

Upon disassembling the generator after the tests were completed, the thermoelements were found in the condition as shown in Figure 14. All of the P-type elements, which are on the bottom row in Figure 14, were chipped or broken, while none of the N-type elements were damaged. The physical characteristics for the two types of elements were listed identically in strength features as shown by Table II. An inquiry to the manufacturer yielded the information that the elements are very weak in shear strength, particularly the P-type elements. It was concluded that as the elements moved outward upon being heated and the cold ends moved farther apart, a shear stress was set up in the elements. The P-type elements could not withstand the stress and broke either in the element material or at the junctions, and the stress in the N-type elements was thereby relieved. Another source of difficulty could be the curved shape of the cold-junction electrodes, which would place more stress on the corners of the elements than at the center. Still another difficulty could be the tight fit of the elements in the holes. Close examination of the N-type elements revealed that they slumped slightly out of shape, which can be observed in Figure 14 as a slight bowing in the long dimension.



**Figure 14. A View of the Thermoelements After the Initial Tests of the Generator With the N-type Elements in the Top Row and the P-type Elements in the Bottom Row**



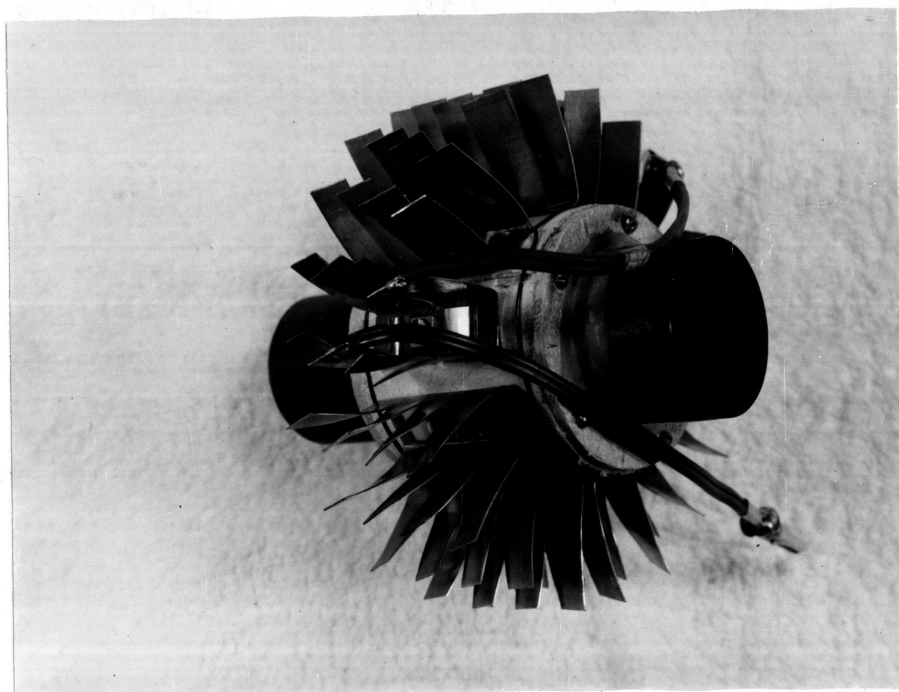
A considerable oxide film development was observed on the surface of the hot-junction electrodes where they contacted the thermoelements. Any future design should incorporate some method of providing a non-oxidizing atmosphere around the hot junctions.

### Redesign Considerations

The results of tests of the initial generator design indicated several areas where additional or revised design considerations were required. Thermal stresses on the thermoelements had to be reduced, some method to seal the hot junctions had to be provided, and the heat transfer rate to the hot junctions had to be increased to obtain a greater voltage output.

Elimination of the curved cold-junction electrodes was accomplished by connecting the element pairs across the rows at the cold junctions as shown in Figure 15. One element from each row forms a couple at the cold junctions and two adjacent elements from the same row form a couple at the hot junctions. This system of series connection required an even number of rows of elements, whereas the previous system required an even number of elements per row.

This system also simplified the method of attaching leads as shown in Figure 15. One electrode was split and the leads were attached to the two separate halves. The hold-down clip for this electrode was insulated from the electrode halves by a small piece of paper gasket material.



**Figure 15. A General View of the Revised  
Thermoelectric Generator**

Another simplification which this type of electrode provided was that only one screw was needed to fasten down each electrode. A spring clip made of 0.020-inch-thick and 1/2-inch-wide feeler-gage stock bent to provide the recommended five-pound compressive loading per element was used to hold the electrode in place. One-half-inch-long size 6-32 screws were self-tapped into holes drilled into the center ring of the generator body to compress the spring clips.

To seal the hot junctions from the air 1/32-inch-thick asbestos gaskets were cut to fit between each asbestos ring. Two additional rings of 1/8-inch-thick asbestos cement board were cut to fit over the pipe with zero clearance. These rings were placed one on each end of the generator with an asbestos gasket cut about 1/16-inch in diameter smaller than the pipe and placed between each outer ring and the generator body. When the entire assembly was pulled together the two undersize gaskets were squeezed between the outer rings and the generator body, resulting in a tight seal around the pipe. The generator assembly was held together by ten size 5-40 screws, 1 1/2-inches long, placed in holes drilled through the outer rings and tapped into the center ring, five from each side. To seal the thermoelements in their holes, an asbestos sheet was wrapped around the generator. The thermoelements were pushed through the asbestos sheet, forming their own holes. This flared the edges of the asbestos down into the 1/32-inch oversize holes in the generator body. This method of sealing left very little air in the generator so that the first few minutes of operation at high temperature were sufficient to burn out the oxygen. No vacuum

or reducing atmosphere was required because of the very small air volume.

The cooling fins and cold-junction electrodes were made of single pieces of copper  $3 \frac{5}{8}$  inches long by 2 inches wide by 0.022 inches thick. The fin portion consisted of  $1 \frac{1}{2}$ -inch flanges bent up on each end leaving a  $\frac{5}{8}$ -inch by 2-inch section in the middle to act as the electrode. Each of the 40 fins was slit radially into four sections for a distance of 1 inch and the sections separated by twisting a few degrees. This afforded a better contact between the fins and the cooling air.

An inverted metal cone was constructed to fit around the exhaust pipe directly under the generator. This cone prevented cooling air from being drawn up along the pipe, which resulted in cooler air and a hotter pipe. In addition, the exhaust pipe was shortened to three inches above the muffler, which further reduced the heat loss to the atmosphere.

Heat-collecting fins were constructed to increase the heat transfer rate to the hot junctions. A  $\frac{1}{16}$ -inch-thick steel sleeve 4 inches long was press-fitted inside the exhaust pipe in the area where the generator was mounted. Four  $\frac{1}{16}$ -inch by  $\frac{3}{4}$ -inch by 4-inch steel fins were brazed to the inner surface of the sleeve so they extended radially in toward the center and divided the interior of the sleeve into four pie-shaped segments. The fins offered a greater surface area to transfer heat from the exhaust gas without an appreciable increase in back-pressure. The sleeve served to distribute the heat uniformly

and to aid conduction from other areas toward the region being reduced in temperature by the transfer of heat through the thermoelements.

### Tests of the Revised Generator Design

Another set of thermoelements was obtained from the manufacturer and another generator was assembled which incorporated these several additions and revisions to the original design. The redesigned generator was then tested to evaluate the changes in design.

The generator with the cooling hood attached was again tested out-of-doors on the same tractor as previously used. The data collected from these tests were recorded in Table VI. The hot-junction temperatures shown were calculated from the cold-junction temperature and the open-circuit voltage by use of the curves in Figure 2.

Adjustment of the fuel mixture and ignition timing to other than the maximum power and economy conditions was necessary to get the maximum exhaust temperature. An allowance should be made for this by designing a system for a hot-junction temperature lower than the maximum of 1100°F so this temperature would not be exceeded if the engine subsequently got out of adjustment.

A large discrepancy existed between the expected internal resistance under low operating temperature conditions and that actually obtained from the generator. For example, from trial five in Table VI the internal resistance was 0.52 ohms. From Figure 4, however, at the same average temperature of 258°F the resistivity of 10 couples was 0.08 ohms. The difference is believed to be due mainly to high contact



TABLE VI. DATA COLLECTED FROM TESTS OF THE FORCED AIR COOLED REVISED  
THERMOELECTRIC GENERATOR ON A TRACTOR ENGINE

Exhaust- Gas Temp. (°F)	Hot- Junction Temp.* (°F)	Cold- Junction Temp. (°F)	Fin Temp. (°F)	Cooling Air Temp. (°F)	Engine Speed (rpm)	Load on Engine (hp)	Open- Circuit Voltage V (volts)	Short- Circuit Current I (amps)	Internal Resistance $R = \frac{V}{I}$ (ohms)	External Power $P = \frac{I(V)}{2}$ (watts)
1294	1061	101	68	18	1900	37	2.30	6.25	0.37	3.60
1330	1129	179	145	76	1900	35	2.25	6.00	0.38	3.38
1105	847	97	70	29	1900	30	1.65	4.00	0.41	1.65
978	763	138	118	80	1600	18	1.23	2.50	0.49	0.77
695	443	73	54	27	1400	10	0.65	1.25	0.52	0.20

\*Estimated from the cold-junction temperature and the open-circuit voltage

resistance at the hot junctions caused by less thermal expansion and a lower compressive loading than at high temperature levels. Also, several cold junctions were found to be loose, which would have a tendency to increase the total resistance, particularly under lower stress values. It was possible that the clearance between the cold-junction electrodes and the generator body was insufficient to allow the spring clips to maintain the thermoelement loading at low exhaust temperatures. This clearance was less at low temperatures due to less thermal expansion of the pipe and thermoelements, and excess pools of solder from the cold junctions may have filled the gap.

There were four loose cold junctions and all of them were at the junctions of the P-type thermoelements. Two of the breaks were in the solder and 2 were in the thermoelement material adjacent to the soldered connections. This breakage was believed to be due to vibration, which was quite severe on the exhaust pipe at high power settings.

In working with both generator designs, it was very difficult to maintain a strong bond between the thermoelement material and the tin-bismuth solder at the cold junctions. A sonic bonding technique is used by the manufacturer to pre-tin the thermoelements, and any conventional means of applying more solder was unsuccessful. The solder applied by the manufacturer had a low melting point (about 200°F) which resulted in very close tolerances within which the cold junctions had to be maintained.

The maximum generator output obtained was 3.6 watts compared to a design power of 5.2 watts. This reduced output was due partly to the less than maximum hot-junction temperature, but it was due mainly to the increased internal resistance of 0.37 ohm instead of the design 0.28 ohms. The design resistance was shown not to be a practical value upon which to calculate an expected power output since it was unattainable with the types of hot and cold junctions used.

## SUMMARY AND CONCLUSIONS

### Summary

A thermoelectric generator was designed and built for operation on the exhaust pipe of an agricultural tractor. The generator contained 10 N-type and 10 P-type lead telluride thermoelements held in an asbestos cement board body. Forced air cooling was provided by a venturi section in the exhaust stream which drew cooling air over fins attached to the cold junctions. The design output was 5.2 watts, or 4.3 amps at 1.2 volts, with a temperature difference of 1000°F across the thermoelements. The performance of the generator was observed during actual operation and the results compared with the expected performance. Faults in the original design were corrected and a second generator was assembled and tested under similar conditions as the initial generator tests.

Tests of the initial generator design determined that a greater heat transfer rate was necessary between the exhaust gas and the hot junctions and that an improved method for providing compressive loading on the elements was necessary to prevent breakage of the thermoelements under thermal stress.

The maximum open-circuit voltage from the initial generator was 1.6 volts as compared to a design voltage of 2.4 volts. The discrepancy was due to insufficient heat transfer through the exhaust pipe at the hot junctions resulting in less than design hot-junction temperature (900°F instead of design 1100°F). This was corrected in the

revised generator design by using a finned insert in the exhaust pipe to increase the effective area.

The P-type thermoelements failed by fracturing during tests of the initial generator. That the cold-junction electrodes did not sufficiently relieve the thermal stress was believed to be the cause of the failure. This condition was corrected by changing the path of electron flow such that flat cold-junction electrodes could be used. In addition, the electrodes were held in place with spring clips which allowed greater deflection of the thermoelements under thermal stress.

Tests of the revised generator yielded a maximum open-circuit voltage of 2.3 volts as compared to a design value of 2.4 volts, but at less than maximum temperature conditions. The voltage output of any individual generator is dependent upon the actual operating temperature conditions at any moment, and the maximum design voltage depends upon how closely the temperature can be controlled to insure against unknowingly exceeding the maximum allowable temperature which the thermoelements can withstand.

The primary fault of the redesigned generator was that the internal resistance was 0.37 ohm instead of the design 0.28 ohm, and the discrepancy was even greater under less than maximum temperature conditions. This limited the maximum available external power to 3.6 watts instead of the design 5.2 watts.

Some difficulty was encountered in maintaining satisfactorily soldered cold junctions. The P-type thermoelements apparently could not withstand the thermal stress and vibration transferred through the



rigidly soldered cold junctions and either separated from the solder or sheared apart in the thermoelement material.

Depending upon the future course of thermoelectric material development, power generation by means of an exhaust-gas thermoelectric generator may or may not become practical. The cost of the materials must certainly decrease. An improved cold-junction joining technique and more durable thermoelement materials must be developed to withstand the vibration, and more long-term information must be collected concerning the expected useful life of thermoelectric materials under various operating conditions.

### Conclusions

Based upon the results of tests conducted on the thermoelectric generators developed, several conclusions were made concerning the practicability of this method of electric power generation. They are as follows:

1. The theory and principles involved in designing an exhaust-gas thermoelectric generator were straightforward and yielded results in general agreement with design values.
2. Since precise temperature control was not possible due to changes in air temperature, load on the engine, and fuel mixture and ignition timing settings, each installation would have to be considered individually to determine the allowance which should be made for these conditions to prevent overheating of the thermoelements.

3. Allowance for an internal resistance of 100 per cent more than theoretical should be made when determining a generator output with the present recommended pressure-type hot junctions.
4. Soldered cold junctions were not sufficiently durable to withstand the vibration and thermal stress encountered in the generator as designed. A better joining method and a higher melting material are needed for making a satisfactory cold junction.
5. The lead telluride thermoelement materials used were brittle and subject to shattering under the conditions of vibration and thermal stress as encountered in the exhaust-gas thermoelectric generator designs developed in this study.

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**APPENDICES**



**APPENDIX A. DATA COLLECTED IN MEASUREMENT  
OF EXHAUST-GAS TEMPERATURES**

Table VII and Table VIII are data collected in measurement of exhaust-gas temperatures by the use of thermocouples. A gasoline tractor engine and a diesel tractor engine were used as representing the principle types of engines used in agricultural work. The points in the tables for which percentages of maximum horsepower at rated pto speed are calculated also are shown in Figure 4 in the text. The other points were of interest for estimating the potential value of each type of engine for producing thermoelectric power at part-load and at other than rated pto speeds.

TABLE VII. EXHAUST GAS TEMPERATURE MEASUREMENT OF A  
CASE "400" GASOLINE TRACTOR

Ambient Air Temperature (°F)	Engine Speed (rpm)	Power Output (hp)	Percent of Maximum Hp*	Thermocouple** Reading (mv)	Exhaust-Gas Temperature (°F)
88	750	0	---	15.70	722
88	1000	5	---	16.20	743
88	1200	0	---	16.58	760
91	1600	15	---	23.05	1034
90	1900	15	40	26.90	1196
89	1900	25	66	29.25	1297
91	1900	35	92	29.70	1316

\*Based on 38 horsepower

\*\*Chromel-Alumel thermocouple

TABLE VIII. EXHAUST GAS TEMPERATURE MEASUREMENT OF AN  
IHC "560" DIESEL TRACTOR

Ambient Air Temperature (°F)	Engine Speed (rpm)	Power Output (hp)	Percent of Maximum Hp*	Thermocouple** Reading (mv)	Exhaust Gas Temperature (°F)
85	600	0	---	4.25	218
85	1000	0	---	4.48	228
85	1200	0	---	4.75	240
85	1000	5	---	5.80	287
85	1300	15	---	7.65	370
85	1750	15	25	8.18	394
85	1750	25	42	9.90	470
85	1750	35	58	12.65	591
85	1750	45	75	16.00	735
85	1750	55	92	18.17	828
85	1750	60	100	24.00	1074

\*Based on 60 horsepower

\*\*Chromel-Alumel thermocouple

## APPENDIX B. THEORETICAL CALCULATIONS OF HEAT TRANSFER AT THE HOT JUNCTIONS

### Exhaust Velocity Measurement

The exhaust-gas velocity was determined by taking velocity-head readings with a pitot tube in the exhaust stream of a gasoline engine under the following conditions: the exhaust pipe inside diameter was 2.0 inches, the engine displacement was 148 cubic inches, the engine speed was 1800 rpm, and the exhaust temperature was 1200°F ( $\rho = 0.024$  lb/ft<sup>3</sup>). The maximum velocity head (h) measured was

$$h = 4.25 \text{ in H}_2\text{O} \left( \frac{1 \text{ lb/in}^2}{2.31 \text{ ft H}_2\text{O}} \right) \left( \frac{\text{ft}^3}{0.024 \text{ lb}} \right) \frac{12 \text{ in}}{\text{ft}}$$

$$h = 920 \text{ ft air}$$

and the maximum exhaust velocity (v) was

$$v = \sqrt{2gh} = \sqrt{64.4 \text{ ft/sec}^2 (920 \text{ ft})}$$

$$v = 244 \text{ fps} = 14,620 \text{ fpm.}$$

Applying a correction factor (c) for the maximum temperature of 1400°F, a rated engine speed of 1850 rpm, and a 1.875-inch inside diameter exhaust pipe to correspond to conditions under which the thermoelectric generator could operate gives a value of

$$c = \frac{(2.0 \text{ in})^2}{(1.875 \text{ in})^2} \times \frac{1850 \text{ rpm}}{1850 \text{ rpm}} \times \frac{1860^\circ\text{R}}{1660^\circ\text{R}}$$

$$c = 1.31.$$

The corrected velocity ( $v_c$ ) was

$$v_c = cv = 1.31 (14,620 \text{ fpm})$$

$$v_c = 19,150 \text{ fpm.}$$

### Hot-Junction Area Requirements

Exhaust gas is made up of approximately 70 per cent nitrogen, 20 per cent carbon dioxide, and 10 per cent water vapor by weight (10). The properties of this mixture are sufficiently similar to air to consider them alike for heat transfer purposes. The pressure within the exhaust pipe was considered to be atmospheric.

It was assumed that one-third of the temperature drop of 3000°F was through the film barrier between the gas and the pipe wall and the other two-thirds was through the pipe wall, the thin cement coating, and the iron hot-junction electrodes. Brown and Marco (3) present a formula for the film coefficient ( $h_c$ ) in BTU/hr ft<sup>2</sup> °F for air which is given as

$$h_c = C \frac{G^{0.8}}{D^{0.2}}$$

where C is a coefficient given in table form for various air temperatures (0.00513 for air at 1400°F), G is the mass velocity ( $\rho V$ ), lb/ft<sup>2</sup> hr, and D is the diameter of the pipe in feet. The value of  $\rho$  for air at 1400°F is 0.0213 lb/ft<sup>3</sup>.

$$G = \rho V = 0.0213 \text{ lb/ft}^3 (19,150 \text{ ft/min}) 60 \text{ min/hr}$$

$$G = 24,500 \text{ lb/ft}^2 \text{ hr}$$

$$D = \frac{1.875 \text{ in}}{12 \text{ in/ft}} = 0.156 \text{ ft}$$

$$h_c = 0.00513 \left( \frac{(24,500)^{0.8}}{(0.156)^{0.2}} \right)$$

$$h_c = 0.00513 \left( \frac{(3260)}{(.689)} \right)$$

$$h_c = 24.3 \text{ BTU/hr ft}^2 \text{ °F.}$$



The heat transfer ( $Q_f$ ) through a surface film where  $t_1$  and  $t_2$  are the hot and cold temperatures on either side of the film is determined from the formula

$$Q_f = h_c A (t_1 - t_2)$$

$$\frac{Q_f}{A} = 24.3 \text{ BTU/hr ft}^2 \text{ } ^\circ\text{F (100}^\circ\text{F)}$$

$$\frac{Q_f}{A} = 2430 \text{ BTU/hr ft}^2.$$

It was earlier determined that 225 BTU/hr were required by 20 elements, which means the required inside pipe surface was

$$A = \frac{225 \text{ BTU/hr}}{2430 \text{ BTU/hr ft}^2} = 0.0926 \text{ ft}^2 = 13.33 \text{ in}^2$$

and the area required per element was

$$A = \frac{13.33 \text{ in}^2}{20 \text{ elements}} = 0.667 \text{ in}^2/\text{element}.$$

The inside circumference (c) of the pipe was

$$c = \pi D = \pi(1.875 \text{ in}) = 5.90 \text{ in}$$

so the length of pipe (L) required by the elements was

$$L = \frac{13.33 \text{ in}^2}{5.9 \text{ in}} = 2.26 \text{ in}.$$

# APPENDIX C. THEORETICAL CALCULATIONS OF HEAT TRANSFER AT THE COLD JUNCTIONS

## Heat Load on the Cooling Fins

The outside radius of the generator body was 1 9/16 inches. This was composed of 1-inch pipe radius, a 1/16-inch-thick hot-junction electrode, and 1/2-inch-long thermoelements. The area ( $A_c$ ) of cooling surface for a 2-inch length of pipe was

$$A_c = \pi DL = \pi (3.125 \text{ in}) 2 \text{ in} = 19.65 \text{ in}^2$$

if the entire area were covered by cold-junction electrodes, which was not practical since the electrode for each couple had to be separate from every other electrode. A value of about 15 square inches of cooling surface without fins appeared to be a more practical value.

Since the generator body was not to contact the electrodes except at the points of support the generator body did not have to be cooled to the 100°F which was desirable for the thermoelements. The heat transfer rate ( $Q_g$ ) through the generator body with a hot side temperature of 900°F and a cool side temperature of 300°F is given by the formula

$$Q_g = KA \, dt/dL_g = KA \frac{(t_h - t_c)}{L_g}$$

where

$K$  = thermal conductivity (BTU/hr ft °F)

= 0.43 BTU/hr ft °F for cement asbestos board

$A$  = cross-sectional area at the mean diameter (ft<sup>2</sup>)

$t_h - t_c$  = temperature difference (°F)

$L_g$  = length of heat travel = generator-body thickness (ft).

The area at the mean diameter was

$$\begin{aligned}
 A &= \pi \left( \frac{D_o + D_i}{2} \right) L - \text{area of elements} \\
 &= \pi \left( \frac{3.00 \text{ in} + 2.25 \text{ in}}{2} \right) 2.0 \text{ in} - 20 \left( \frac{\pi}{4} \right) \left( \frac{1}{4} \text{ in} \right)^2 \\
 &= 16.5 \text{ in}^2 - 0.98 \text{ in}^2 \\
 &= 15.52 \text{ in}^2 \\
 A &= 0.108 \text{ ft}^2
 \end{aligned}$$

and the heat transfer when the thickness ( $L_g$ ) = 0.375 inches = 0.0312 feet was

$$Q_g = 0.43 \text{ BTU/hr ft } ^\circ\text{F} (0.108 \text{ ft}^2) \left( \frac{900^\circ\text{F} - 300^\circ\text{F}}{0.0312 \text{ ft}} \right)$$

$$Q_g = 892 \text{ BTU/hr.}$$

If it were assumed that one-half of the heat transferred through the generator body was carried away directly by the cooling air and the other one-half was added to that transferred through the thermoelements, the total heat to be dissipated from the electrodes would be 446 BTU/hr plus 190 BTU/hr (transferred by thermoelement conduction) plus 17 BTU/hr (transferred by Peltier effect), or 653 BTU/hr.

#### Generator Hood and Venturi Design

The jet-pump principle was used to create a flow of cooling air across the cooling fins. A jet pump performs by ejecting fluid at high velocity into a venturi section of a tube. This great increase in velocity creates a pressure differential between the venturi throat and the supply tube, and fluid flows through the tube toward the region of

reduced pressure and into the high velocity stream. Jet-pump theory is involved and would only apply to conventional venturi forms with carefully formed surfaces and transition shapes. Also, any venturi system must be calibrated even when theoretical principles are closely adhered to. For these reasons no attempt was made to determine the optimum shape of hood or the air flow it should create.

A hood with a simplified venturi section was built as in Figure 7 and was tested to determine the air flow rate it produced. The hood was built to allow 1 1/2-inch-long fins to be placed on the cold-junction electrodes, and was so designed to be easily moved up or down approximately one inch as a means of regulating the air flow rate.

From preliminary tests conducted with the hood placed over the exhaust pipe of a gasoline tractor engine, cooling air velocities of 800 fpm at idle to 2000 fpm at full power were obtained. The maximum velocity was used to design the cooling fin area required, since at lower velocities the quantity of heat to be dissipated was also reduced.

#### Required Cooling Fin Area

Brown and Marco (3) give a relationship for the surface coefficient for turbulent flow of air parallel to plane surfaces, which is

$$h_c = 0.055 \frac{K}{L_f} \left( \frac{L V^e}{\mu} \right)^{0.75}$$

where

$h_c$  = surface coefficient (BTU/hr ft<sup>2</sup> °F)

$L_f$  = length of surface (ft)

$K$  = thermal conductivity of air (BTU/hr ft °F)

$V$  = fluid velocity (ft/hr)

$\rho$  = fluid density (lb/ft<sup>3</sup>)

$\mu$  = dynamic viscosity (#/ft hr).

For air at 70°F and one-inch-wide fins,

$$h_c = 0.055 \frac{(0.015)}{(0.0833)} \left( \frac{(0.0833)(2000)(60)(0.075)}{0.045} \right)^{0.75}$$

$$h_c = 0.0099 (1460) = 14.5 \text{ BTU/hr ft}^2 \text{ °F.}$$

The temperature drop between the fins and the air was assumed as 50°F. Previously the heat to be dissipated from the electrodes was determined to be 653 BTU/hr. Thus, the necessary fin area was

$$A = \frac{Q}{h_c \Delta t} = \frac{653 \text{ BTU/hr}}{(14.5 \text{ BTU/hr ft}^2 \text{ °F}) 50 \text{ °F}}$$

$$A = 0.900 \text{ ft}^2 = 129.5 \text{ in}^2.$$

A fin length (L) of 1 1/2 inches and a width (w) of 1 inch were chosen as representing the largest practical fin size. The number of fins (n) required with two cooling surfaces per fin was

$$n = \frac{A}{2 L w} = \frac{129.5 \text{ in}^2}{2 (1.5 \text{ in})(1.0 \text{ in})} = 43 \text{ fins.}$$